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A CLASS OF SEQUENTIAL INPUT ADAPTIVE SYSTEMS

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David E. Greene, Major, USAF



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NOTICES

This final report was submitted by personnel of the Biomathematics Modeling Branch, Data Sciences Division, USAF School of Aerospace Medicine, Aerospace Medical Division, AFSC, Brooks Air Force Base, Texas, under job order 2312V519.

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This report has been reviewed by the Office of Public Affairs (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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A CLASS OF SEQUENTIAL INPUT ADAPTIVE SYSTEMS

INTRODUCTION

The sequential input adaptive system theory introduced in a previous publication (2) is generalized to n^{th} order systems in this paper. A comparative study is made of the first, second, and third order systems.

In the new theory it is assumed that a living controller, through conditioning, accommodates to the input and the controlled plant so that the total system has basic mathematical properties. In this controller-centered theory, predictions are made instantaneously and tracking movements are governed by the strategy.

The sequential theory describes and predicts manual control tracking behavior in a complex antiaircraft artillery (AAA) man-machine system (2). For this system, the sequential theory gives a description of mean tracking behavior that has a closer correlation with the experimental data than does the description given by the optimal control approach of Kleinman et al. (4). Furthermore, the sequential theory applies to the human eye tracking system (3).

In this paper the basic optimal process is first presented. This process is then sequentially applied to represent first, second, and third order input adaptive systems. Finally, the theoretical tracking descriptions are compared with tracking data from an AAA man-machine system.

THE BASIC OPTIMAL PROCESS

The basic optimal process is defined by an n^{th} order differential equation in projected error with parameters determined such that a cost functional is minimized. Properties of the basic optimal process are presented for first, second, and third order systems.

The input adaptive system is represented in Figure 1.

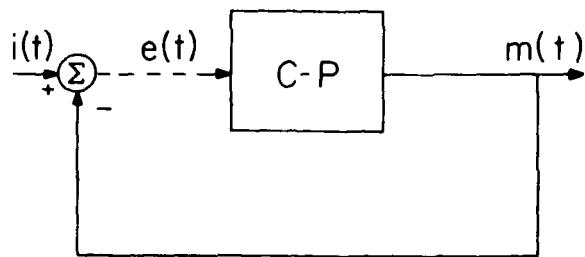


Figure 1. The input adaptive system.

The internal system C-P is the controller-plant. It is assumed that in the n^{th} order system the controller

1. estimates the system error state at discrete times t_i ($t_0 < t_1 < \dots < t_m$) of the tracking interval

$$e(t_i) \equiv i(t_i^+) - m(t_i^-) \quad (1)$$

•
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$$e^{(n-1)}(t_i) \equiv i^{(n-1)}(t_i^+) - m^{(n-1)}(t_i^-),$$

2. predicts the input at each t_i by n terms of a Taylor series representation of the input, and

3. effects systematic open-loop control over the intervals (t_i, t_{i+1}) to reduce the error relative to the predicted input.

There is no time delay in the prediction-control process. The input is assumed to be piecewise $n-1$ times differentiable and such that the one-sided limits in equation 1 exist.

The prediction-control process is represented by the repeated application of a basic optimal process. In the basic optimal process, the system input $i(t)$ is predicted at time t_i by

$$\hat{i}(t) = \sum_{j=0}^{n-1} \frac{i(j)(t_i^+)}{j!} (t - t_i)^j \quad t > t_i \quad (2)$$

and an error response to this predicted input, $\hat{e}(t) \equiv \hat{i}(t) - m(t)$ where $m(t)$ is the system output, is determined by

$$\hat{e}^{(n)}(t) + \sum_{j=1}^n \alpha_j \hat{e}^{(n-j)}(t) = 0 \quad t > t_i \quad (3)$$

$$\hat{e}^{(j)}(t_i) = i(j)(t_i^+) - m(j)(t_i^-) \quad 0 \leq j \leq n-1$$

where (α_j) are constants that minimize the associated cost functional

$$J_n = \int_{t_i}^{\infty} \left[(\hat{e}^{(n)}(t))^2 + \sum_{j=1}^n (\beta_j \hat{e}^{(n-j)}(t))^2 \right] dt \quad (4)$$

for given nonnegative constants (β_j) with $\beta_n > 0$. The constants (β_j) are denoted the strategy.

Justification for the forms in equations 3 and 4 follows from that given in reference 2. The basic operation in defining the variational problem is the extension of the upper limit in the cost functional and the continuation of the projected error through all future time.

In the basic optimal process for $n = 1, 2$, and 3 , the parameters (α_j) are uniquely determined by the strategy (β_j). Cost functional J_1 has a unique minimum when

$$\alpha_1 = \beta_1. \quad (5)$$

Cost functional J_2 has a unique minimum when

$$\alpha_1 = (\beta_1^2 + 2\beta_2)^{1/2} \quad (6)$$

$$\alpha_2 = \beta_2. \quad (7)$$

Cost functional J_3 has a unique minimum when

$$\alpha_1 = (p+A+B)^{1/2} + \left\{ 2p - (A+B) + 2 \left[(p - \frac{1}{2}(A+B))^2 + \frac{3}{4}(A-B)^2 \right]^{1/2} \right\}^{1/2} \quad (8)$$

$$\alpha_2 = (\hat{p}+\hat{A}+\hat{B})^{1/2} + \left\{ 2\hat{p} - (\hat{A}+\hat{B}) + 2 \left[(\hat{p} - \frac{1}{2}(\hat{A}+\hat{B}))^2 + \frac{3}{4}(\hat{A}-\hat{B})^2 \right]^{1/2} \right\}^{1/2} \quad (9)$$

$$\alpha_3 = \beta_3 \quad (10)$$

where

$$A = \left[-\frac{b}{2} + \left(\frac{b^2}{4} + \frac{a^3}{27} \right)^{1/2} \right]^{1/3}$$

$$B = \left[-\frac{b}{2} - \left(\frac{b^2}{4} + \frac{a^3}{27} \right)^{1/2} \right]^{1/3}$$

$$p = \frac{1}{3} \beta_1^2$$

$$a = \frac{1}{3} \left(3\beta_2^2 - \beta_1^4 \right)$$

$$b = \frac{1}{27} \left(-2\beta_1^6 + 9\beta_1^2\beta_2^2 - 27\beta_3^2 \right)$$

and

$$A = \left[-\frac{\hat{b}}{2} + \left(\frac{\hat{b}^2}{4} + \frac{\hat{a}^3}{27} \right)^{1/2} \right]^{1/3}$$

$$\hat{B} = \left[-\frac{\hat{b}}{2} - \left(\frac{\hat{b}^2}{4} + \frac{\hat{a}^3}{27} \right)^{1/2} \right]^{1/3}$$

$$\hat{p} = \frac{1}{3} \beta_2^2$$

$$\hat{a} = \frac{1}{3} \left(3\beta_1^2\beta_3^2 - \beta_2^4 \right)$$

$$\hat{b} = \frac{1}{27} \left(-2\beta_2^6 + 9\beta_1^2\beta_2^2\beta_3^2 - 27\beta_3^4 \right).$$

The roots of complex numbers $z = r e^{i\theta}$ in equations 8 and 9 are taken as

$$z^{1/n} = r^{1/n} \left[\cos \left(\frac{\theta}{n} \right) + i \sin \left(\frac{\theta}{n} \right) \right].$$

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Critical points for J_1 , J_2 , and J_3 may be obtained by direct computation (evaluation and differentiation of the cost functional) or by other variational methods (1). The cost functionals may be shown to have minimums at the critical points defined by equations 5-10 through a consideration of quadratic forms (see reference 5).

Cost functionals J_1 , J_2 , and J_3 , evaluated for unit step inputs applied at $t = 0$ with zero initial conditions on the system output, are given for reference in the Appendix.

The strategy parameters (β_j) have physical meaning: they indicate the importance the controller gives to each quantity in equation 4. The manner in which the strategy affects the system response is clearly illustrated in the second order system when the input is a step function. For this input, the system output response is overdamped when $\beta_1 \gg \beta_2$ (so that $\beta_1^2 \gg 2\beta_2$) and is underdamped when $\beta_2 \gg \beta_1$ (so that $\beta_1^2 \ll 2\beta_2$).

Two additional properties of the variational problems are given. First, any solution to equation 3 generated by the variational process tends to zero as t becomes large. This follows from the existence of the cost functional (equation 4). Second, the relationships that minimize J_n ($n = 2$ or 3) also minimize J_{n-1} ; for if β_n is set equal to zero in these relationships, then the parameters (α_j) that minimize J_{n-1} are obtained. (For $n = 3$, this is most easily shown using the necessary conditions for J_3 to have a minimum as given in the Appendix.)

In the basic optimal process the input is predicted by n terms of a Taylor series representation of the input. An error response to the predicted input is determined by the strategy and the initial conditions through an n^{th} order differential equation.

THE ADAPTIVE PROGRAM

The basic optimal process is sequentially applied in the tracking algorithm called the adaptive program. In option 1) of the adaptive program the basic optimal process is applied at constant increments of time Δt_c . In option 2) of the adaptive program the basic optimal process is applied when the absolute value of the system error exceeds the system error threshold E_T , but is applied such that $t_{i+1} - t_i \geq T_S$ where T_S is the minimum period for a sequential problem. (The criteria in option 2) can be generalized to include threshold conditions on derivatives of the system error.) The adaptive program predicts the input over successive time intervals by a sequence of steps, ramps, or parabolas (for $n = 1, 2$, or 3 , respectively) and produces n th order error responses (dependent upon the strategy) to the components of the predicted input. The system output and the system error are defined over each subinterval (t_i, t_{i+1}) by $m = \hat{i} - \hat{e}$ and $e = i - m$, respectively.

CONTINUOUS MODELS

Suppose that the input $i(t)$ is n times differentiable for $t > t_0$ and the strategy is constant. If the time intervals between successive applications of the basic optimal process are small, then the sequence of differential equations 3, solved in the adaptive program, is approximated by

$$e^{(n)}(t) + \sum_{j=1}^n \alpha_j e^{(n-j)}(t) = i^{(n)}(t) \quad t > t_0 \quad (11)$$

$$e(t_0) = i(t_0) - m(t_0)$$

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$$e^{(n-1)}(t_0) = i^{(n-1)}(t_0) - m^{(n-1)}(t_0).$$

This result uses the identity $\hat{e} = \hat{i} - i + e$. The initial value problem (11) is an approximate continuous representation for the sequential theory.

Associated with equation 11 is a feedback control system with unity feedback and open-loop transfer function $\sum_{j=1}^n \alpha_j s^{-j}$. This feedback control system is another continuous model.

Equations 5-10 qualitatively describe how the strategy (β_j) regulates the parameters (α_j) in these models.

The transition from a higher order system to a lower order system is examined for the special case where the input is n times differentiable for $t > t_0$. Consider the system (equation 11) or the associated feedback control system and $n = 2$ or 3 . Let (α_j) be formally defined in terms of (β_j) by equations 6-10. As β_n tends to zero, the steady-state solution to the n th order system approaches the steady state solution to the $n-1$ th order system with the same input. (This is most easily seen using Laplace transforms.) Therefore, there is a regular transition in the steady-state solution through the strategy from a higher order system to a lower order system when the input is properly differentiable.

ANTIAIRCRAFT ARTILLERY MAN-MACHINE TRACKING SYSTEM

The sequential theory is applied to manual control tracking in an AAA system. The theoretical tracking descriptions provided by the first, second, and third order input adaptive systems are compared with the experimental data.

The tracking experiment is described in a previous publication (2). Two well-trained operators manually tracked targets on a simulated AAA system.

One operator controlled the system in azimuth and the other operator controlled the system in elevation.

In the present comparative study, only the Trajectory I tracking task is considered and only option 1) of the adaptive program is used. The azimuth and elevation components of Trajectory I are given in Figure 2. Time histories of ensemble averages (15-18 runs) of the experimental azimuth and elevation tracking errors are given in Figure 3.

In the analysis it is assumed that the strategy of each operator is constant throughout the tracking task. The adaptive program is applied separately to each azimuth and elevation tracking task. For convenience, the initial conditions used in the adaptive program are $\hat{e}(0) = \dots = \hat{e}^{(n-1)}(0) = 0$.

The parameters used in the adaptive program for the second order system are those which were previously identified from the AAA tracking data (2). For the purpose of comparison, the β_n component of the strategy is held fixed for the first, second, and third order systems. The parameters used in the adaptive program for each n^{th} order system are

$$\beta_j = \begin{cases} 1 & 1 \leq j < n \\ 10 & j = n \end{cases}$$

and $\Delta t_c = .1$ s. For each system the strategy is primarily to minimize the projected error.

The adaptive program system tracking errors on the azimuth and elevation components of Trajectory I are given in Figure 4 for the first, second, and

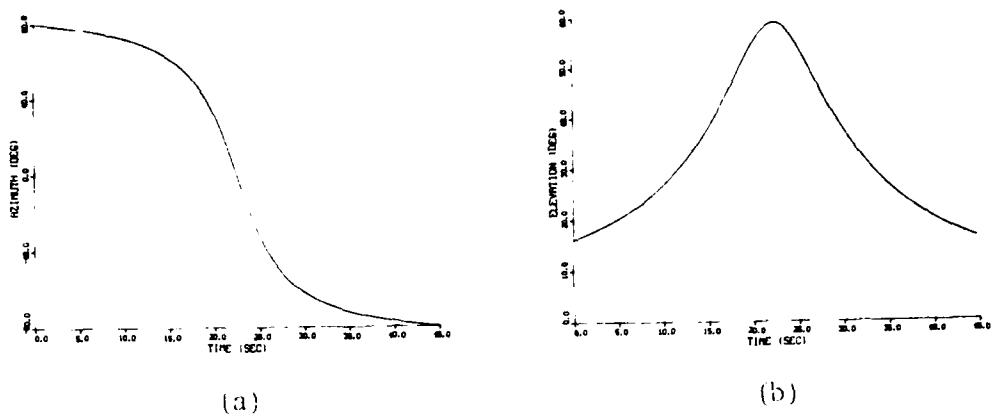


Figure 2. Trajectory I components: (a) azimuth, (b) elevation.

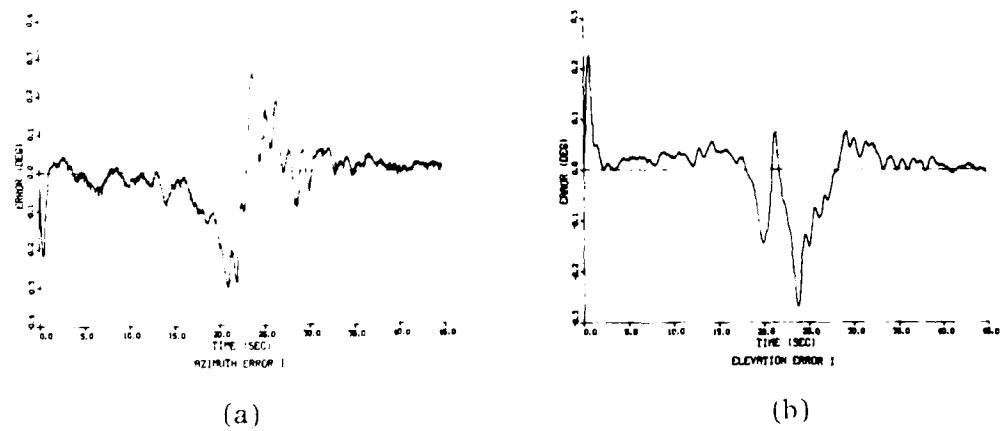


Figure 3. Experimental mean tracking errors on the azimuth and elevation components of Trajectory I: (a) azimuth, (b) elevation.

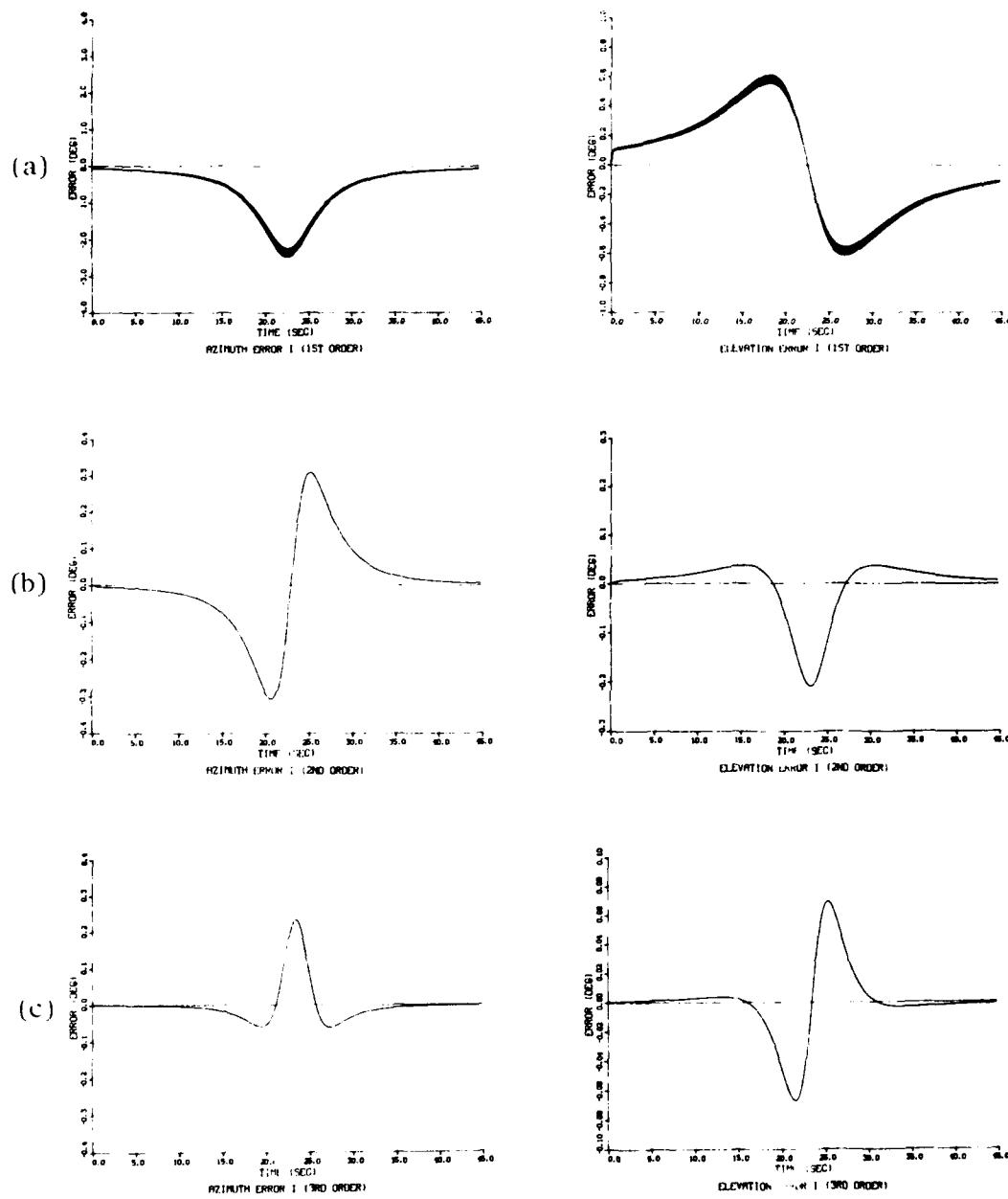


Figure 4. Adaptive program option 1) tracking errors on the azimuth and elevation components of Trajectory I: (a) first order system, (b) second order system, (c) third order system.

third order systems. The option 1) tracking errors for the given strategies become smaller as n increases. This reduction in tracking error is consistent with the improved predictions of the input (equation 2) as n increases. The n th order system tracking errors are proportional to the n th derivatives of the input. The second order system tracking errors are characteristic of the experimental data. As is shown in reference 2, the option 1) second order system tracking errors on all trajectories have a closer correlation with the experimental data than those of the optimal control approach (4).

The first order system with its single strategy parameter does not represent the tracking behavior in the AAA system. First, the first order system tracking errors considerably differ in amplitude and profile from the experimental tracking errors. Second, the identified parameter β_2 in the second order system is such that $\beta_2 = \alpha_2 \gg 0$ and therefore the tracking response is characteristic of at least a second order system.

Properties of the continuous model (equation 11) are used to classify the AAA tracking response. Although the tracking responses in Figure 4 were computed as sequences of transient responses, they essentially represent steady-state solutions to equation 11. (When the exact initial conditions are used or when the input is discontinuous, the transient response may dominate the steady-state response (2).) As noted earlier, there is a regular transition in the steady-state solution through the strategy from a higher order system to a lower order system. Therefore, a third order system response corresponding to strategy (1, 10, ϵ) where ϵ is positive and sufficiently small approximates the second order system response corresponding to strategy (1, 10) (provided $i(t)$ is properly differentiable). This demonstrates that a third order system can also represent the tracking behavior in the AAA system.

Further information is known about the AAA system. The subjects were trained to minimize mean-square tracking error (2). This suggests that the β_3 component of the strategy should be large. Thus, a third order system with strategy (1, 10, ϵ) and ϵ positive and small is not appropriate. It is further noted that a third order system with small β_3 strategy component would be ineffective in reducing the tracking error associated with the sudden, initial appearance of the target. In addition, third order system descriptions with $\beta_3 \gg \beta_1$ and $\beta_3 \gg \beta_2$, as illustrated in Figure 4, are not characteristic of the tracking data.

The second order system is the lowest order system of this class that describes the mean tracking behavior in the AAA system. The second order system description with strategy $\beta_2 \gg \beta_1$ is consistent with the manner the subjects were trained.

DISCUSSION

A mathematical theory was presented for first, second, and third order sequential input adaptive systems. In these systems, a theoretical controller predicts the input at instants by n terms of a Taylor series representation of the input and effects open-loop control over intervals to obtain n^{th} order projected error responses.

The second order system is the lowest order system of this class that produces a description characteristic of the mean tracking behavior in the AAA man-machine system. Higher order system descriptions may not be appropriate in this application. In a third order system, for example, the controller would have to regularly infer the target acceleration from the visual display and control using this information, stressing or exceeding human capabilities. Third order system descriptions with small β_3 strategy component are not appropriate because the subjects were trained to minimize mean-square

tracking error. Therefore, the second order system is the most plausible system consistent with limitations of the human controller and the manner the subjects were trained.

A distinctive feature of the new theory is that it represents the overall system tracking response by a sequence of transient responses. The sequential theory provides a broad nonlinear description for tracking systems with living controllers.

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APPENDIX

Cost functionals J_1 , J_2 , and J_3 , evaluated for unit step inputs applied at $t = 0$ with zero initial conditions on the system output, are

$$J_1 = \frac{1}{2\alpha_1} \left[\alpha_1^2 + \beta_1^2 \right]$$

$$J_2 = \frac{1}{2\alpha_1\alpha_2} \left[\alpha_2^3 + \beta_1^2\alpha_2^2 + \beta_2^2 \left(\alpha_1^2 + \alpha_2^2 \right) \right]$$

$$J_3 = \frac{1}{2\alpha_3(\alpha_1\alpha_2 - \alpha_3)} \left[\alpha_3^3\alpha_2 + \beta_1^2\alpha_3^3 + \beta_2^2\alpha_1\alpha_3^2 + \beta_3^2 \left(\alpha_1^2\alpha_3 + \alpha_1\alpha_2^2 - \alpha_2\alpha_3 \right) \right].$$

Necessary conditions for J_3 to have a minimum are

$$\alpha_3 = \beta_3$$

with α_1 and α_2 such that

$$\alpha_2^2 + \alpha_2 \left(\beta_1^2 - \alpha_1^2 \right) + 2\alpha_1\beta_3 + \beta_2^2 = 0$$

$$\alpha_1^2\beta_3 + \alpha_1 \left(\beta_2^2 - \alpha_2^2 \right) + 2\alpha_2\beta_3 + \beta_3\beta_1^2 = 0.$$